Some results on Gaussian beta ensembles at high temperature

Trinh Khanh Duy

Institute of Mathematics for Industry, Kyushu University, Japan

trinh@imi.kyushu-u.ac.jp

1. Gaussian beta ensembles

Gaussian beta ensembles (G E). Ensembles of real points with joint density

$$({}_1; : : : ; {}_n) \propto \prod_{i<j} | {}_j \; {}_i| \; e^{\frac{1}{2}({}_1^2 + \; + {}_n^2)} d \; :$$

They are generalizations of GOE, GUE and GSE, and can also be viewed as the equilibrium meas. of a one dim. Coulomb log-gas at the inverse temperature .

Jacobi matrix models (Dumitriu and Edelman 2002).

$$T_{n;} = \begin{pmatrix} N(0;1) & \tilde{}_{(n-1)} & & \\ \tilde{}_{(n-1)} & N(0;1) & \tilde{}_{(n-2)} & \\ & \ddots & \ddots & \ddots \\ & & \tilde{} & N(0;1) \end{pmatrix};$$

where $\begin{cases} N(\; ; \; ^2): \text{ Gaussian distribution with mean and variance } ^2; \\ \tilde{}_k = \frac{1}{\sqrt{2}} \; {}_k = \sqrt{\text{Gamma}(\frac{k}{2};1)}. \end{cases}$

The eigenvalues $({}_1; : : : ; {}_n)$ of $T_{n;}$ are distributed as G E.
Let $q_j = |v_j(1)|$, where $v_1; : : : ; v_n$ are the corresponding normalized eigenvectors. Then $(q_1^2; : : : ; q_n^2)$ have Dirichlet distribution with parameter $\frac{}{2}$, and are independent of $({}_1; : : : ; {}_n)$.

We are interested in the following quantities.

1. Empirical distribution/measure: $L_{n;} = \frac{1}{n} \sum_{j=1} \; {}_j$.
2. Spectral measure (to be de ned on the right): ${}_{n;} = \sum_{j=1} q_j^2 \; {}_j$.
3. Bulk statistics: ${}_n = \sum_{j=1} {}_{n(\, {}_j \; E)}$; for xed E.

2. Spectral measures of Jacobi matrices

Jacobi matrices (nite and in nite):

$$J = \begin{pmatrix} a_1 & b_1 & & \\ b_1 & a_2 & b_2 & \\ & \ddots & \ddots & \ddots \\ & & b_{n-1} & a_n \end{pmatrix}; \quad J = \begin{pmatrix} a_1 & b_1 & & \\ b_1 & a_2 & b_2 & \\ & b_2 & a_3 & b_3 \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

A

;