



Department of  
Economics and Finance

Working Paper No. 17-10

Economics and Finance Working Paper Series

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Testing the Fisher Hypothesis in the G7  
Countries Using I(d) Techniques

May 2017

<http://www.brunel.ac.uk/economics>



## **1. Introduction**

to be 0 or 1 and allows it instead to take any real value; this is clearly a much more general specification allowing for a richer dynamic structure. In particular, consider the following model:

$$R_t = \alpha + \beta \pi_{t+1} + \epsilon_t, \quad (1)$$

where  $R_t$  is the nominal interest rate and  $\pi_t$  is the inflation rate (under the implicit assumption of rational expectations, i.e.,  $\pi_{t+1} = E_t \pi_{t+1}$ ).

If both  $R_t$  and  $\pi_t$  are I(0) variables, standard regression methods can be applied. On the other hand, if they are I(1) a cointegration approach becomes necessary. Unlike in the classical case, we allow these variables to be I(d) with  $0 < d < 1$  or  $d > 1$ . First we estimate  $d_R$  and  $d_\pi$ , i.e., their respective orders of integration, then we make the series stationary I(0) by differencing them to obtain  $\tilde{R}_t = (1 - B)^{d_R} R_t$

## **2. The Fisher Effect: A Brief Literature Review**

Early studies analysed the Fisher effect without considering stationarity issues. These include Fama (1975),

process. Further evidence of long-memory behaviour in interest rates is provided by Barkoulas and Baum (1997), Meade and Maier (2003), Gil-Alana (2004a,b), Couchman, Gounder and Su (2006), Gil-Alana and Moreno, 2012, Haug, 2014, Apergis et al., 2015, Abbritti et al. (2016), etc. As for inflation rates, evidence of long memory has been reported in many papers including Hassler (1993), Delgado and Robinson (1994), Hassler and Wolters (1995), Baillie et al. (1996), Baum et al. (1999), Hyung et al. (2006), Kumar and Okimoto (2007), etc. Lardic and Mignon (2003) found some evidence for the Fisher hypothesis in the G7 countries using semi-parametric  $I(d)$  techniques based on log-periodogram regressions. The opposite conclusion was reached by Ghazalia and Ramlee (2003) by estimating fully parameterised AutoRegressive Fractionally Integrated Moving Average (ARFIMA) models for the same set of countries. Kasmanmet et al. (2006) examined the Fisher relationship with fractional cointegration techniques in 33 developed and developing countries. They found no evidence of cointegration when using classical methods (i.e., Johansen, 1996); however, they found fractional cointegration by using the Geweke and Porter-Hudak (1982) approach on the estimated errors from the cointegrating relationship. Similar conclusions were reached in the case of Turkey by Burcu (2013) and for Nigeria by Etuk et al. (2014).

### **3. The Empirical Methodology**

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where  $y_t$  is the original series (in our case,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^{d_0} x_t = u_t; \quad t = 1, 2, \dots, \quad (7)$$

with uncorrelated and Bloomfield (1973) errors in turn. In the case of inflation rates, a time trend is required only for the US with autocorrelated errors,



5, despite the evidence from Table 3, both variables are assumed to be  $I(0)$ , and therefore  $d_R = d_I = 0$ , while in Table 6 the estimated values from Table 3 are used for  $d_R$  and  $d_I$ .

In Table 4, we impose  $d_R = d_I = 1$  - this is a common assumption in the empirical literature that usually treats interest rates and inflation as being non-stat

**[Insert Table 7 about here]**

Finally, in Table 7, we compare the estimates of  $\beta$  with those obtained when imposing  $d = 0$  in (6). They are generally higher than in the previous cases, especially when  $d_R = d_I = 0$ . In fact, in the  $I(0)$  case, even the hypothesis  $\beta = 1$  cannot be rejected for some countries (France, Germany and Italy). However, it should be noted that this specification is incorrect since the null hypothesis of  $d = 0$  is rejected in favour of  $d > 0$  in all the cases shown in Table 5.

Overall, the evidence based on our preferred model (Table 6ii) suggests that there exists a positive relationship between nominal interest rates and inflation, since the  $\beta$  coefficient is positive; however, it is statistically different from 1 (more precisely, it is smaller). Therefore, we do not find evidence of the full adjustment of nominal rates to inflation required by the Fisher hypothesis.

## **5. Conclusions**

This paper revisits the Fisher hypothesis in the G7 countries using fractional integration and cointegration models that are more general than the standard ones based on the classical  $I(0)$ / $I(1)$  dichotomy. Two sets of results are produced under the alternative assumptions of uncorrelated and autocorrelated errors. The univariate analysis suggests that the differencing parameter is higher than 1 for most series in the former case, whilst the unit root null cannot be rejected for the majority of them in the latter case. The multivariate results imply that there exists a positive relationship, linking nominal interest rates to inflation; however, the Fisher hypothesis is rejected since there is no full adjustment of the former to the latter. The implication of our findings is that the evidence in favour of the Fisher effect found in various studies is invalidated by their failure to take into account the fractional nature of the series of interest as well as of the regression errors.





Hyung, N., P.H. Franses and J. Penm (2006). Structural breaks and long memory in US inflation rates. Do they matter for forecasting?, *Research in International Business and Finance* 20, 1, 95-110.

Johansen, S. (1996). *Likelihood based inference in cointegrated vector autoregressive models*, Oxford University Press.

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**Table 1: Estimates of d and 95% confidence bands for the inflation series**

i) Uncorrelated (white noise) errors		
No deterministic terms	An intercept	A linear time trend

**Table 2: Estimates of d and 95% confidence bands for the interest rate series**

i) Uncorrelated (white noise) errors			
	No deterministic terms	An intercept	A linear time trend
CANADA	1.11 (0.98, 1.26)	<b>1.20 (1.03, 1.42)</b>	1.20 (1.02, 1.42)
FRANCE	1.12 (1.01, 1.27)	<b>1.23 (1.06, 1.46)</b>	1.23 (1.06, 1.46)

**Table 3: Estimates of the orders of integration for each series**

	Inflation rates	Interest rates
CANADA	<b>0.73 (0.43, 1.11)</b>	<b>0.72 (0.41, 1.13)</b>
FRANCE	<b>1.18 (0.88, 1.52)</b>	<b>0.73 (0.49, 1.06)</b>
GREAT BRITAIN	<b>1.12 (0.86, 1.54)</b>	<b>0.77 (0.50, 1.13)</b>
GERMANY	<b>1.18 (0.93, 1.56)</b>	<b>0.80 (0.53, 1.17)</b>
ITALY	1.40 (1.08, 1.77)	<b>0.93 (0.79, 1.16)</b>
JAPAN	<b>0.99 (0.76, 1.30)</b>	0.20 (0.03, 0.48)
U.S.A.	<b>0.70 (0.42, 1.06)</b>	<b>0.71 (0.49, 1.12)</b>



**Table 4: Estimates of  $d$ ,  $\alpha$  and  $\beta$  in the long run equilibrium relationship using  $d = 1$**

	$d$ (and 95% conf. band)	$\alpha$	$\beta$	# (t-
i) Uncorrelated (white noise) errors				





**Table 7: Estimates of  $\alpha$  and  $\beta$  in equations (5) and (6) with  $d$  imposed to be equal to 0**

	$d_R = d_I = 1$		$d_R = d_I = 0$		$d_R$ and $d_I$ estimated	
	#	\$	#	\$	#	\$
CANADA	-0.0170 (-1.30)	<b>0.0544</b> <b>(1.75)</b>	<b>2.2081</b> <b>(33.49)</b>	<b>0.3354</b> <b>(9.57)</b>	<b>0.0483</b> <b>(1.66)</b>	<b>0.4248</b> <b>(7.30)</b>
FRANCE	-0.0182 (-1.31)	<b>0.1383</b> <b>(2.44)</b>	<b>1.7197</b> <b>(34.14)</b>	<b>0.8459***</b> <b>(25.92)</b>	<b>0.0479</b> <b>(1.89)</b>	<b>0.5828</b> <b>(6.90)</b>
GR. BRITAIN	-0.0198 (-1.38)	<b>0.1578</b> <b>(3.47)</b>	<b>2.1473</b> <b>(29.40)</b>	<b>0.4250</b> <b>(15.62)</b>		