

Department of Economics and Finance



**STOCK MARKET INDICES AND INTEREST RATES
IN THE US AND EUROPE:
PERSISTENCE AND LONG-RUN LINKAGES**

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Abstract

This paper uses fractional integration/cointegration methods to analyse (i) the persistence of the S&P500 and DAX stock indices as well as of the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in the US and Europe respectively. The data are monthly

1. Introduction

The aim of this paper is to examine the degree of persistence of some representative interest rate and stock price series for the US and Europe as well as the possible existence of long-run equilibrium linkages between these two variables in each case. More specifically, the two interest rate series used for the empirical analysis are the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, whilst the stock indices are the S&P500 and the German DAX; the former includes the 500 stocks with the largest market cap that are traded in the US and covers a wide variety of sectors: information and technology (Oracle, Microsoft, Mastercard), health care (Johnson & Johnson), financial (JPMorgan Chase & Co., Berkshire Hathaway), consumer discretionary (Starbucks), etc.; the latter comprises 40 companies with German headquarters chosen on the basis of their market cap as well as liquidity conditions.

The Fed's Effective Federal Funds rate is the interest rate charged to banks when they lend money to each other overnight (it is also known as the overnight rate), whilst the ECB's Marginal Lending Facility rate is the rate banks pay when they borrow from the ECB overnight (a collateral being required). Therefore in both cases an interest rate rise will decrease profitability by making debt more expensive and thus reducing the capital available for investment;

Kuttner (2005) concluded that the effects of unanticipated monetary policy actions on expected excess returns account for the largest part of the response of stock prices.

Note that causality could also run in the opposite direction. For instance, Rigobon and Sack (2003) used an identification method based on heteroscedasticity and reported that a 5 percent rise (fall) in the S&P 500 index increased the likelihood of a 25 basis point tightening (easing) by the Fed by about a half. Hashemzadeh and Taylor (1988) carried out the Granger–Sims test and also found that causality runs from interest rates to stock prices. Bjørnland and Leitemo (2009) estimated a Vector AutoRegressive (VAR) model and found bidirectional causality between the S&P500 and the Federal Funds rate.

The present study aims to shed further light on the behaviour of interest rates and stock indices, as well as their possible linkages,

use the adjusted closing price (the results are almost the same using the closing price instead). The interest rate series have been obtained from the FRED webpage

$$\text{---} + \text{.} \quad (3)$$

As already mentioned, the t(W*BT/F9 12 Tf1 0 0 1 313.97 755.88 Tm0 g0 G{0003>ITJ5eW*BT/F2 12

- i) If x_{1t} (stock prices) and x_{2t} (interest rates) are both integrated of a given order, say d , and then
- ii) Regressing each stock price series on the corresponding interest rate series,

And testing if the estimated residuals are integrated of a smaller order, i.e., $d - b$, with $b > 0$, which would imply cointegration (see Engle and Granger, 1987, and more recently Cheung and Lai, 1993, and Gil-Alana, 2003).

4. Empirical Results

As a first step we carry out ADF, Phillips and Perron (1988), Kwiatkowski et al. (1992) or Elliot et al. (1996) unit root tests, all of which imply that the series are nonstationary.

(these results are not reported for reasons of space). HnJETQq0.000008871 0 595.32 841.92 reW*nBT/F4

$d = 1$ cannot be rejected, which represents evidence in favour of the Efficient Market Hypothesis (EMH). For the ECB rate the estimated value of d is 1.45 with a confidence interval of (1.36, 1.57), and for the Fed rate it is 1.56 with a confidence interval of (1.48, 1.66), and thus the null of $d = 1$

Table 5: Estimates of the coefficients in the regression model

Regression model	Intercept (t-value)	Regr. Coefficient (t-value)
S&P500 / FED	3.2575 (212.67)	-0.0209 (-3.59)
DAX / ECB	4.0340 (316.99)	-0.0741 (-16.54)

Table 6: Estimates of d for the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
i) White noise errors			
S&P500 / FED	1.08 (1.01, 1.17)	1.08 (1.01, 1.16)	1.08 (1.01, 1.16)
DAX / ECB	1.12 (1.04, 1.23)	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)

of frequencies around the origin. An improved version of the test for the stationary case is given in Christensen and Nielsen (2006).

In the two-variable case, the NBFDSL estimator proposed in Robinson (1994) is given by:

$$\hat{\beta} = \frac{1}{m} \sum_{j=1}^m \operatorname{Re} I_{y_1 y_1}^{-1} \frac{1}{m} \sum_{j=1}^m \operatorname{Re} I_{y_1 y_2} \quad (5)$$

which is asymptotically distributed as:

$$\sqrt{m} (\hat{\beta} - \beta) \xrightarrow{D} N \left(0, \frac{g_e^{-1} 2d^2}{2g_{y_1}^{-1} 2d 2d_e} \right) \quad (6)$$

where g_e and g_{y_1} are the elements of a diagonal matrix. From (6), normality is ensured as long as $m \rightarrow \infty$ (Christensen and Nielsen, 2006). Note that this estimator crucially depends on the value of the bandwidth parameter m .

Table 7: Estimates of d in the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
S&P500 / FED			
i) White noise errors			
m = 0.5	0.99 (0.91, 1.08)	1.05 (0.99, 1.13)	1.05 (0.99, 1.14)
m = 0.6	0.97 (0.90, 1.06)	0.95 (0.89, 1.05)	0.95 (0.88, 1.05)
m = 0.7	0.99 (0.91, 1.08)	1.05 (0.98, 1.13)	1.05 (0.98, 1.13)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	1.00 (0.86, 1.17)	1.08 (0.96, 1.22)	1.09 (0.96, 1.23)
m = 0.6	1.00 (0.86, 1.14)	1.08 (0.96, 1.11)	1.09 (0.97, 1.11)
m = 0.7	1.00 (0.86, 1.17)	1.09 (0.97, 1.22)	1.09 (0.97, 1.23)
DAX / ECB			
i) White noise errors			
m = 0.5	1.00 (0.93, 1.10)	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)

m = 0.6	0.96 (0.89, 1.06)	1.04 (0.96, 1.14)	1.04 (0.96, 1.14)
m = 0.7	1.00 (0.93, 1.10)	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	0.99 (0.87, 1.15)	1.13 (0.98, 1.33)	1.13 (0.98, 1.32)
m = 0.6	0.95 (0.84, 1.13)	0.96 (0.84, 1.12)	0.97 (0.83, 1.12)
m = 0.7	0.99 (0.87, 1.16)	1.13 (0.98, 1.33)	1.13 (0.98, 1.33)

Table 7 reports the results based on this estimator, again for the three cases of no regressors, an intercept only, and an intercept as well as a time trend, for three different bandwidth parameters, $m = 0.5, 0.6$ and 0.7 . In all cases the estimates are again very close to 1 and the unit root null hypothesis cannot be rejected, which again provides evidence against (fractional) cointegration.

In the cointegration analysis it is implicitly assumed that all variables are stochastic. In what follows we depart from this assumption by assuming

5. Conclusions

This paper has used fractional integration/cointegration methods to analyse (i) the persistence of the S&P500 and DAX stock indices as well as of the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in both the US and Europe. The data are monthly and the sample period goes from January 1999 to December 2022.

The results can be summarised as follows. All series examined are nonstationary: stock prices are found to be $I(1)$ while interest rates display orders of integration substantially above 1, and therefore all four series are highly persistent, and mean reversion does not occur in any case. Moreover, the fractional cointegration analysis suggests that stock prices and interest rates are not linked in the long run.

Future work should extend the analysis in two ways. First, a multivariate model including other relevant variables such as inflation, money supply, exchange rates etc. should be estimated to shed further light on the linkages between interest rates and stock prices. Second, expectations and announcement effects should be incorporated into the model. It is well known that stock prices can react to anticipated interest rate changes or monetary announcements even before these take place. Because investors have already discounted those changes the observed correction at the time of their implementation will then be smaller, and so will be the estimated impact. Therefore, not allowing for expectation and announcement effect could result in underestimating the strength of the linkages between monetary policy and stock markets.

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