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**PERSISTENCE IN THE REALIZED BETAS:
SOME EVIDENCE FOR THE SPANISH STOCK MARKET**

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Abstract

This paper examines the stochastic behaviour of the realized betas within the one-factor CAPM for the six companies with the highest market capitalization included in the Spanish IBEX stock market index. Fractional integration methods are applied to estimate their degree of persistence at the daily, weekly and monthly frequency over the period 1 January 2000 – 15 November 2018 using 1, 3 and 5-year frequency and time span (number of observations).

Keywords: Realized beta; CAPM; persistence; mean reversion; long memory

JEL Classification: C22; G11

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1. Introduction

The one-factor capital asset pricing model (CAPM), initially introduced in the 1960s, is based on the idea that systematic risk is determined by the covariance between market and individual stock returns and is still the standard framework taught in finance courses and used by risk-averse investors for selecting optimal portfolios. Fama and MacBeth (1973) estimated this model to analyse the relationship between risk and return in NYSE stocks and documented a positive linkage between average return and market beta in the period 1926-1968; however, Fama and French (1992) found that this linear relationship had disappeared in the period 1963-1990.

The one-factor model has several limitations and is based on rather restrictive assumptions (see Fernandez, 2015, 2019); for instance, it requires investors to have homogeneous expectations (of returns, volatility and correlations for every security, over the same time horizon). In its standard formulation it is a linear regression, whose most critical parameter to be estimated is beta, which measures the risk arising from exposure to market-wide as opposed to idiosyncratic factors; polls are instead used to predict market risk, and the yield curve for the expected return of the risk-free asset.

Betas are normally predicted using historical data on the assumption that their future behaviour will be similar. Out of 150 finance textbooks we have reviewed 80 recommend some estimation method but differ in terms of the frequency (daily, weekly, monthly or annual) and the span of data (from 6 months to 25 years) used for this purpose. As in Campbell et al. (1997), we found that the most common estimation approach (in 64% of the cases) is to use monthly data over a 5-year period. However, more recently, higher frequency data have often been used as developments in IT have made computations easier. Table 1 summaries our findings concerning the frequency

and the number of observations (time span) chosen for estimating the realized betas in the textbooks reviewed.

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Among more recent studies focusing on higher frequency data, Andersen et al. (2003) and Bollerslev et al. (2009) analysed intraday trading with samples of 15 minutes. Damodaran¹ on his public portal for beta estimation selected different time periods (5 years and 2 years with weekly returns). Papageorgiou et al. (2016) analysed daily returns over a one-year period and showed that these results outperform those obtained using monthly data over a 5-year period as in Fama and MacBeth (1973). Cenesizoglu et al. (2016) evaluated the accuracy of one-month-ahead beta forecasts (at the monthly, daily and 30-minute frequency) and found that low (high) frequency returns produce the least (most) accurate estimates. Sharma (2016) analysed the conditional variance of various stock indices over 14 years. Bollerslev et al. (2016) investigated how individual stock prices respond to market price movements and jumps using data at the 5-minute intraday frequency with one-year samples, and found evidence that betas associated with intraday discontinuous and overnight returns entail significant risk premiums, while the intraday continuous betas do not. Cenesizoglu et al. (2018) used a realized beta estimator for daily returns over the previous year for 1, 3, and 6-month holding periods to explain momentum effects.

An appropriate estimation period and sampling frequency are clearly crucial for obtaining accurate beta forecasts. An important issue is the possibility of time variation in the betas (Andersen et al., 2003), which is not considered by the standard, one-factor CAPM. Multi-factor pricing models including additional empirically motivated factors, such as

shown to have better in-sample fit and to produce more accurate out-of-sample predictions, but are often criticized because of the difficulty in interpreting the expanded set of variables in terms of systematic risk.

An interesting question in this context is how persistent the betas are. Andersen et al. (2005) apply fractional integration methods to analyse data for 25 Dow Jones Industrial Average (DJIA) stocks over the period 1962-1999 and conclude that the corresponding realized betas are not very persistent and are best modelled as $I(0)$ processes. The present paper uses a similar modelling framework but focuses instead on the Spanish stock market and provides evidence on the degree of persistence of the betas for six companies included in the IBEX index. In contrast to Andersen et al. (2005), we find evidence of persistence, though the results are sensitive to the choice of frequency and time span (number of observations). The layout of the paper is as follows: Section 2 provides a brief literature review; Section 3 outlines the fractional integration model used for the analysis; Section 4 describes the data and discusses the empirical results; Section 5 offers some concluding remarks.

2. Literature Review

In this section we discuss in turn each of the three main approaches to modelling and forecasting the realized betas that have been adopted in the CAPM literature.

2.1 Realized variance and data filtering

A first group of studies focuses on realized variance, covariance, and data filtering. Ghysels and Jacquier (2006) proposed a mix of existing data-driven filters and parametric methods. Hooper et al. (2008) compared a series of competing models to forecast beta; specifically, the applied realized measures of asset return variances and

covariances following the methodology proposed in Andersen et al. (2005). Christoffersen et al. (2008) used the information embedded in the prices of stock options and index options to compute the forward-looking market beta at the daily frequency, using option data for a single day. Chang et al. (2012) found

information. Ang and Chen (2007) proposed a conditional CAPM with time-varying betas and market risk premia.

In the last decade, additional factors have been considered. Garleanu and Pedersen (2011) introduced the margin-

sample forecasting method for monthly market returns using the Variance Risk Premium (VRP) defined in Bollerslev et al. (2009) as the difference between the objective and the risk-neutral expectations of the forward variance. Bai et al. (2019) proposed a general equilibrium model to quantify the consumption CAPM performance. Hollstein et al. (2019) proposed a link between conditional betas and high high-frequency data to explain asset pricing anomalies.

2.3 Long memory in asset pricing

A third approach introduced by Bollerslev et al. (1988) focuses on long-run dependence. Following the early contribution of Robinson (1991), many subsequent studies showed the empirical relevance of long memory for asset return volatility (e.g., Ding et al., 1993). Robinson (1995) developed a formal framework for testing long-run dependence in the logarithmic volatilities; the FIGARCH model was used by Baillie et al. (1996) to analyse exchange rates, and by Bollerslev and Mikkelsen (1996) to examine US stock market, in both cases long memory being detected, with the series being modelled as mean-reverting fractionally integrated processes, where the conditional variance decreases at a slow hyperbolic rate. Andersen and Bollerslev (1997) concluded that long memory is an intrinsic feature of returns. Bollerslev and Mikkelsen (1999) provided evidence of mean reversion in the volatility process using fractionally integrated models.

Cochran and DeFina (1995) found predictable periodicity in market cycles. Bollerslev and Mikkelsen (1996) concluded that long-run dependence in the US stock market is best modelled as a mean-reverting fractionally integrated process. However, Andersen and Bollerslev (1997) found that this process is very slow for most returns, and thus detecting mean reversion is not an easy task. Balvers et al. (2000) pointed out

proposed a GARCH model incorporating realized measures of variances and covariances. Engle (2016) put forward the Dynamic Conditional Beta (DCB) model to estimate regressions with time-varying parameters.

A brief comparison between the most popular market beta estimation techniques can be found in Hollstein and Prokopczuk (2016), who examined the performance of several time-series models and option-implied estimators, and suggested using the hybrid methodology of Buss and Vilkov (2012) since it consistently outperforms all other approaches.

3. Methodology

We analyse persistence in the realized betas by using fractional integration methods to estimate the degree of dependence in the data, which is measured by the differencing parameter d . For our purposes we define a covariance stationary process $\{x_t, t = 0, \pm 1, \dots\}$ as integrated of order 0, and denote it by $I(0)$, if the infinite sum of its autocovariances is finite. This type of processes, also known as short-memory ones, include not only the white noise but also the stationary and invertible ARMA-type of models. To generalise, we can define the process $\{y_t, t = 0, \pm 1, \dots\}$ as integrated of order d , and denote it by $I(d)$, if d -differences are required to make it $I(0)$, i.e.,

implying that

$$(1 - B)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots$$

The parameter d plays a crucial role in this context, since it is a measure of the degree of persistence of the series: the higher is d , the higher is the degree of dependence between observations. More specifically, $d = 0$ implies short memory behaviour, while $0 < d < 0.5$ characterises a covariance stationary long-memory process; if $0.5 \leq d < 1$, the series is non-stationary but mean-reverting with shocks having long-lasting effects that disappear in the long run; finally, $d = 1$ implies non-stationarity and lack of mean reversion.

Although fractional integration was already proposed in the early 1980s by Granger (1980, 1981), Granger and Joyeux (1989) and Hosking (1981), it was not until the late 1990s and early 2000 that it became popular in economics and finance (Baillie, 1996; Gil-Alana and Robinson, 1997; Mayoral, 2006; Gil-Alana and Moreno, 2012; Abbritti et al., 2016; etc.). We estimate the differencing parameter using the Whittle function in the frequency domain (Dahlhaus, 1989) by using a version of the LM tests of Robinson (1994) which is computationally very attractive.

4. Data and Empirical Results

We have obtained data on daily, weekly and monthly returns from the Reuters Eikon database for the six companies with the highest market capitalization included in the IBEX-35 (ISIN ES0SI0000005), the most popular Spanish stock index, over the period 1 January 2000 – 15 November 2018. Specifically, we consider the following six companies: BBVA (ISIN ES0113211835), Santander (ISIN ES0113900J37), Telefonica (ISIN ES0178430E18), Inditex (ISIN ES0148396007), Endesa (ISIN ES0130670112) and Iberdrola (ISIN ES0144580Y14). Using the raw data, we construct daily, weekly

and monthly realized beta series by applying the formula _____ and selecting 1, 3 and 5-year samples. Thus, we calculate 9 beta measures for each company.

The estimated model is the following

MacBeth (1973) “standard” beta measure (based on 5 years of monthly observations), estimates of d significantly higher than 1 (which imply lack of mean reversion) are found in the case of BBVA (1.06), Endesa (1.13) and Inditex (1.04), while weak evidence of mean reversion (values of d significantly below 1) is obtained for the cases of Iberdrola (0.96), Telefonica (0.96) and Santander (0.97). However, at the weekly or daily frequency, in all cases but one (Telefonica, 5-year span, weekly observations) mean reversion does not occur. With a 5-year span and daily observations, the estimated values of d range from 1.04 to 1.12, while in the case of a 5-year span and weekly observations the corresponding range is [1.05 - 1.10], except in the case of Telefonica (0.85), as already mentioned.

By contrast, the results based on a 1-year span and monthly observations suggest the presence of mean reversion, the estimates of d ranging from 0.85 to 0.96, except in the case of BBVA (1.08). These estimates should be seen as less reliable because of the smaller sample size on which they are based, and clearly show how crucial the choice of frequency, span and sample size are for estimation purposes.

INSERT TABLE 3 ABOUT HERE

Table 3 displays the estimated values of d under the assumption of weak autocorrelation for the error term. In this case, the only significant regressor is the intercept. Mean reversion is found with a 1-year span and monthly data, with the estimates of d in the range [0.77 - 0.97], whilst the opposite holds when using a 5-year span (with daily, weekly and monthly observations). The range for the estimated values for d is narrower in the case of daily observations [1.15 - 1.25], compared to weekly [1.00 - 1.19] and monthly [1.12 - 1.26], which clearly reflects the respective sample sizes.

To summarise, we find evidence of non-stationary behaviour, with orders of integration equal to or higher than 1, in the vast majority of cases, with mean reversion ($d < 1$) occurring only in a few cases. By contrast, as previously mentioned, Andersen et al. (2005) had concluded that the realized quarterly betas from daily returns in the US over the years 1962-1999 had a lower order of integration than the market variance, with d ranging between 0 and 0.25 for the individual stocks and between 0.35 and 0.45 for the market as a whole; higher degrees of integration were found for the monthly realized betas with 15-minute intraday trading during the years 1993-1999.

The differences between our findings and those reported by Andersen et al. (2005) can be explained if one considers, firstly, that our study focuses on the Spanish market during the period 2000-2018, more specifically on 6 stocks representing 51.8% of the total market capitalization and thus a much larger percentage of the IBEX-35 than the corresponding one for the 30 US stocks from the SP-500 analysed by Andersen et al. (2005) over the period 1962-1999. Secondly, those authors used daily returns for estimating the betas over 3-month periods, while we have used a much longer span of data, from 1 to 5 years. Thirdly, unlike Andersen et al. (2005) we do not pre-filter the data. However, consistently with their study, we also find lower values of d for shorter time spans.

Our results highlight the importance of the choice of frequency and time span (number of observations) for estimation purposes. In particular,

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Table 1: Estimation of the realized betas: chosen frequency and number of observations (time span) in finance textbooks

Daily Weekly

Table 2: Estimates of d with white noise errors

BBVA		No deterministic terms	An intercept	Intercept and time trend
Daily	1y	1.03 (1.01, 1.05)	1.09 (1.07, 1.10)	1.09 (1.07, 1.10)
	3y	1.00 (0.98, 1.03)	1.08 (1.06, 1.10)	1.08 (1.06, 1.10)
	5y	1.00 (0.97, 1.03)	1.06 (1.04, 1.09)	1.06 (1.04, 1.09)
Weekly	1y	1.03 (0.99, 1.09)	1.02 (0.97, 1.07)	1.02 (0.97, 1.07)
	3y	1.01 (0.97, 1.07)	1.06 (1.01, 1.11)	1.06 (1.01, 1.11)
	5y	1.00 (0.95, 1.06)	1.10 (1.05, 1.14)	1.10 (1.05, 1.14)
Monthly	1y	1.02 (0.91, 1.15)	1.08 (0.97, 1.22)	1.08 (0.97, 1.22)
	3y	1.01 (0.92, 1.12)	1.11 (1.01, 1.23)	1.11 (1.01, 1.23)
	5y	1.01 (0.90, 1.16)	1.06 (0.96, 1.18)	1.06 (0.96, 1.18)

ENDESA		No terms	An intercept	Intercept and time trend
Daily	1y	1.02 (1.00, 1.05)	1.04 (1.02, 1.05)	1.04 (1.02, 1.05)
	3y	1.01 (0.98, 1.03)	1.07 (1.05, 1.09)	1.07 (1.05, 1.09)
	5y	1.01 (0.98, 1.04)	1.12 (1.09, 1.14)	1.12 (1.09, 1.14)
Weekly	1y	1.05 (1.00, 1.10)	1.05 (1.00, 1.11)	1.05 (1.00, 1.11)
	3y	1.03 (0.99, 1.08)	1.05 (1.01, 1.09)	1.05 (1.01, 1.09)
	5y	1.01 (0.97, 1.07)	1.05 (1.01, 1.10)	1.05 (1.01, 1.10)
Monthly	1y	0.92 (0.81, 1.05)	0.89 (0.79, 1.03)	0.89 (0.79, 1.03)
	3y	1.10 (1.01, 1.23)	1.16 (1.06, 1.29)	1.16 (1.06, 1.29)
	5y	1.12 (1.03, 1.25)	1.13 (1.03, 1.27)	1.13 (1.03, 1.27)

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