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LONG MEMORY AND FRACTIONAL INTEGRATION IN HIGH FREQUENCY DATA ON THE US DOLLAR / BRITISH POUND SPOT EXCHANGE RATE

Guglielmo Maria Caporale[a]
Brunel University, London

Luis A. Gil-Alana[b]
University of Navarra

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Abstract

This paper analyses the long-memory properties of a high-frequency financial time series dataset. It focuses on temporal aggregation and other features of the data, and how they might affect the degree of dependence of the series. Fractional integration or I(d) models are estimated with a variety of specifications for the error term. In brief, we find evidence that a lower degree of integration is associated with lower data frequencies. In particular, when the data are collected every 10 minutes there are several cases with values of d strictly smaller than 1, implying mean-reverting behaviour; however, for higher data frequencies the unit root null cannot be rejected. This holds for all four series examined, namely Open, High, Low and Last observations for the US dollar / British pound spot exchange rate and for different sample periods.

Keywords: High frequency data; long memory; volatility persistence; structural breaks.
JEL Classification: C22, F31

[a] **Corresponding author.** Department of Economics and Finance, and Centre for Empirical Finance, Brunel University, London, UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. *Email:* Guglielmo-Maria.Caporale@brunel.ac.uk

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dependence the zero and the cyclical frequencies can solve at least to some extent the problem of misspecification that might arise with respect to these two frequencies.

However, the fractional differencing parameter may be very sensitive to the data frequency used in the analysis. In fact, it has often been claimed that aggregation is

Barkoulas and Caglayan (1999) estimated fractional ARIMA (ARFIMA) models for real exchange rates in the post-Bretton Woods era and found almost no evidence to support long run PPP. Additional studies on exchange rate dynamics using fractional integration are those by Crato and Ray (2000), Wang (2004), Dufrenot et al. (2006, 2008) and Aloy et al. (2011) among others. All these papers, however, focus on low frequency (mainly quarterly) data, and do not examine the case of high frequency (intra-day) data.

The present study focuses on the case of spot exchange rates with the aim of gaining some insights into the interaction between fractional integration and high frequency data. The results suggest that lower degrees of memory are associated with lower data frequencies. The layout of the paper is as follows. Section 2 describes the econometric methodology used. Section 3 provides details of the data and discusses the empirical results. Section 4 summarises the main findings and offers some concluding remarks.

2. Methodology

There are two definitions of long memory, one in the frequency domain and the other in the time domain. Let us consider a zero-mean covariance stationary process $\{x_t, t = 0, \pm 1, \ldots\}$ with autocovariance function $\gamma_u = E(x_t x_{t+u})$. The time domain definition of long memory states that:

$$\sum_{u=-\infty}^{\infty} |\gamma_u| = \infty .$$

Assume that x_t has an absolutely continuous spectral distribution, so that it has a spectral density function, f(λ); according to the frequency domain definition of long memory the spectral density function is unbounded at some frequency λ in the interval $[0, \pi)$, i.e.,

$$f(\lambda) \rightarrow \infty \quad as \quad \lambda \rightarrow \lambda^*, \qquad \lambda^* \in [0, \pi].$$

Most of the existing empirical literature considers the case when the singularity or pole in the spectrum occurs at the zero frequency. This is the standard case of $I(d)$ models of the form:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \ldots, \tag{1}$$

with $x_t = 0$ for $t \leq 0$, and $d > 0$, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$, being defined in this context

Robinson (1994) allowing to test for any real value of d in I(d) models. This method is a Lagrange Multiplier (LM) procedure and is the most efficient one in the context of fractional integration. It tests the null hypothesis H_o: $d = d_o$ for any real value d_o, and the test statistic follows a standard (normal) limit distribution. Moreover this standard limit behavior holds independently of the inclusion of deterministic terms (like intercept samples of one day and a half on the basis of the computational time required when working with high frequency data in the context of long memory and fractional integration. However, we also conducted the analysis for a longer time series containing about 4,000 observations (or roughly a week) and the results were once again completely in line with those reported here. We examine four series: Open, High, Low and Last values of the exchange rate collected every minute, where High (Low) stands for the highest (lowest) price and Open (Last) for the initial (last) price observed in that time interval. The aim is to detect whether there exist anomalies in the behaviour of each of the series.

[Insert Figures 1 – 4 about here]

Figure 1 shows plots of the four series. The corresponding returns, obtained as the first differences of the log-prices, are shown in Figure 2. Figures 3 and 4 display the correlograms and the periodograms of the return series. The values of the former seem to indicate that the original series may be I(1), suggesting the possibility of random walk behaviour; however, the presence of some significant values in the correlograms of the first differenced (logged) data, even at lags far away from zero might indicate weak autocorrelation and/or where y_t is the time series observed, α and β are the deterministic terms (an intercept and a linear time trend respectively), and x_t is assumed to be I(d), where d can be any real number. Different assumptions will be made about the error term u_t in (2).[5]

[Insert Table 1 about here]

Table 1 displays the results of the Whittle estimates of d along with the 95% confidence interval of the non-rejection values according to Robinson's (1994) parametric approach. The error term u_t is assumed to be a white noise in Table 1a, an AR(1) process in Table 1b, whilst it is specified using the exponential spectral model of Bloomfield (1973) in Table 1c. The latter is a non-parametric approach to modelling I(0) terms that produces autocorrelations decaying exponentially as in the AR(MA) case.

Table 1 shows the results of the estimated values of d, for the three standard cases of no regressors (i.e., α = β = 0 in (2)), an intercept (α unknown and β = 0), and an intercept with a linear time trend (α and β unknown). Starting with the case of white noise errors (Table 1a), it can be seen that for "Open" and "Last" the estimates are slightly below 1, though the unit root null cannot be rejected in any case. However, for "High" and "Low" the unit root hypothesis is rejected in favour of higher degrees of reversion. When adopting the more general Bloomfield specification (Table 1c), the unit root null hypothesis is never rejected, clearly suggesting that the returns series are I(0). The t-values imply that the time trend coefficients are not statistically significant, whilst the intercepts are always significant. Thus, the model with an intercept seems to be the most adequate specification for these series.

[Insert Figures 5 and 6 about here]

Next we focus on the variance of the return series and examine the squared and absolute returns, which are used as proxies for volatility. These two measures have been widely employed in the financial literature to measure volatility.[6] Plots of the absolute return series are displayed in Figure 5, while Figure 6 shows the squared returns. No structural breaks are apparent in any of these figures.

[Insert Table 2 about here]

Table 2 reports the estimates of d for the absolute and squared returns under the assumption that the error term is white noise. Very similar results were obtained imposing weakly autocorrelated errors. The estimates are significantly positive in all cases, the values ranging between 0.142 (“La

In the context of high frequency data, it is interesting to investigate if the same result holds as the distance between observations increases. For this purpose we examine again the long memory property of the same variables but now using data which are collected every 2, 3, 5 and 10 minutes respectively.

[Insert Table 3 about here]

Table 3 displays the results using these lower frequencies assuming that the error in the relationship between data frequency and the order of integration in the volatility processes is also found in the case of autocorrelated errors.

Finally, we employ a semiparametric method to estimate the values of d for the series of interest, without assuming a functional form for the error term. We follow a procedure developed by Robinson (1995). This method is essentially a local ‘Whittle estimator’ in the frequency domain, which uses a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{s=1}^{m} \log \lambda_s \right), \qquad (3)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{s=1}^{m} I(\lambda_s)\, \lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{T}, \qquad \frac{m}{T} \rightarrow 0,$$

integration of the series.[7] It can be seen that the values are similar for the four series. Along with the estimates we also present the 95% confidence band corresponding to the I(1) hypothesis. We display the estimates for a range of values of the bandwidth parameter m, first from m = 1, …, 100, and th

Fama, 1970), but mean reversion is often found (see, e.g., Poterba and Summers, 1988). More recently, it has become clear that it is essential to consider the possibility of fractional integration in order to analyse the long-memory properties and to allow for a much richer dynamic specification. Various models have been suggested, increasingly general (see, e.g., Caporale and Gil-Alana, 2002, 2007, 2008). The first contribution of the present study is to show that indeed exchange rates dynamics may incorporate long memory components. A potentially crucial issue which has been overlooked is the extent to which the fractional differencing parameter might be sensitive to the data frequency. The second contribution of this paper is to examine this issue empirically using high frequency data on the US dollar-British pound spot exchange rate. In particular, we examined intra-day data (collected every 1, 2, 3, 5 and 10 minutes) for the open, close, high and low values of the exchange rate. In brief, we find evidence that a lower degree of integration is associated with lower data frequencies, and this holds for all the sample periods examined. In particular, when the data are collected every 10 minutes there are several cases with values of d strictly smaller than 1, implying a certain degree of mean-reverting behaviour; however, for higher data frequencies the unit root null cannot be rejected. This is the case for all the four series examined, and for different periods within the sample.

The above results indicate that the order of integration of a time series observed at different intervals may differ. There is no an obvious argument to justify this result. One possibility could be that the data generating process changes with the sampling frequency, although Hassler (2011) showed that this argument is not very strong as in the case of nonstationary fractional integration many of the basic time series properties are preserved under skip sampling. A more plausible argument could be the existence of a bias in the estimation results. Here we have two potential biases. One arises from temporal aggregation as suggested by Souza and Smith (2002).[10] A second type of bias may arise from the high frequency of the data. It is well known that in this case there is microstructure noise and that this noise component becomes stronger as the sampling frequency increases. In the context of the semiparametric log-periodogram estimator Sun and Phillips (2003) derived an explicit form for this bias, which is negative and increases in absolute value as the variance of the noise increases. Sun and Phillips (2004) conjecture that the bias for Whittle-type estimators should be similar. Thus, we have two potential forces that move the bias in opposite directions. Which bias dominates in the case of the methods employed in the present study will be examined in future papers.

In essence, the results suggest that series that are expected to be I(1) consistently with market efficiency might not be so if the sampling frequency is high. Thus, for the 10-minute data, the unit root hypothesis is rejected in favour of mean reversion, Although this does not necessarily imply that the market is inefficient, since the assumption of a random walk is merely a sufficient but not a necessary condition for the EMH.[11]

Finally, it might be asked whether the lower degrees of dependence observed at the lower frequencies is the result of small sample bias. However, it should be noted that even at the lowest data frequencies the sample size is large enough to justify the estimation of a fractional integration model. Extending the dataset to longer periods of time produced very similar results at a high computational cost (all computations were obtained using Fortran and the codes of the programs are available from the authors upon request). Other approaches could be applied to these and other high frequency data

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[10] These authors investigated this bias for parametric and semiparametric estimates of d and showed that the estimates decreased to zero as the sampling frequency decreases.

[11] Note that some of the findings in this paper suggest that the absolute and the squared returns are I(d) with d positive and small, which makes the assumption of a constant variance also questionable.

such as the one suggested by Ohanissian et al. (2008) in their study on fractional integration, structural breaks and data frequency. Note that fractional integration and

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Figure 1: Series in levels

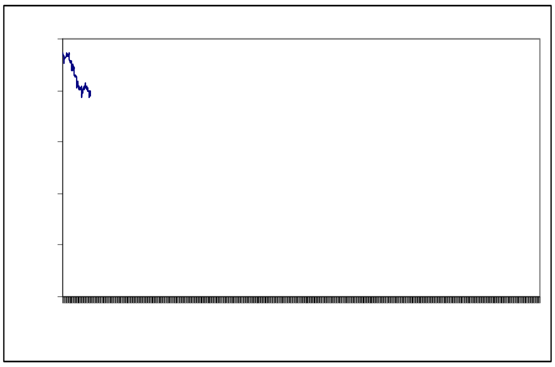
Open	High
	

Table 1: Estimates of the fractional differencing parameter d

a) White noise errors			
	No regressors	An intercept	A linear time trend
Open	0.997 (0.970, 1.027)	0.983 (0.955, 1.015)	0.983 (0.956, 1.015)
High	0.998 (0.971, 1.028)	1.101 (1.066, 1.141)	1.101 (1.066, 1.141)
Low	0.997 (0.970, 1.027)	1.130 (1.095, 1.169)	1.130 (1.095, 1.169)
Last	0.998 (0.970, 1.028)	0.977 (0.950, 1.007)	0.977 (0.950, 1.007)
b) AR (1) errors			
	No regressors	An intercept	A linear time trend
Open	1.381 (1.328, 1.441)	0.973 (0.923, 1.031)	0.974 (0.924, 1.031)
High	1.382 (1.329, 1.442)	0.934 (0.879, 0.996)	0.936 (0.883, 0.996)
Low	1.381 (1.327, 1.440)	0.969 (0.907, 1.037)	0.970 (0.910, 1.037)
Last	1.382 (1.329, 1.442)	1.004 (0.954, 1.060)	1.004 (0.955, 1.060)
c) Bloomfield-type errors			
	No regressors	An intercept	A linear time trend
Open	0.997 (0.944, 1.041)	0.963 (0.922, 1.029)	0.970 (0.923, 1.029)
High	0.991 (0.950, 1.042)	0.962 (0.914, 1.006)	0.962 (0.915, 1.006)
Low	0.990 (0.951, 1.047)	0.988 (0.939, 1.047)	0.988 (0.940, 1.047)
Last	0.998 (0.950, 1.049)	1.010 (0.955, 1.057)	1.010 (0.955, 1.057)

The values in parentheses give the 95% confidence band for the non-rejection values of d.

Figure 5: Absolute returns

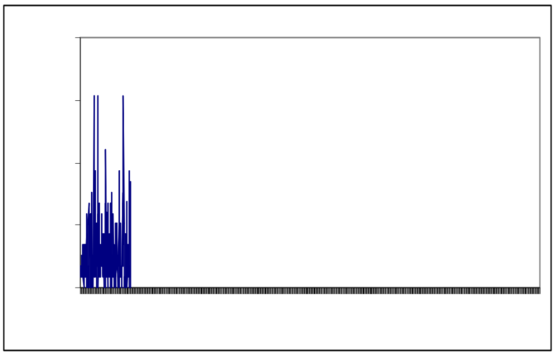
Open	High
	

Table 2: Estimates of d for the absolute and squared returns with white noise errors

a) Absolute returns			
	No regressors	An intercept	A linear time trend
Open	0.149 (0.131, 0.171)	0.148 (0.130, 0.168)	0.144 (0.126, 0.165)
High	0.162 (0.142, 0.185)	0.159 (0.140, 0.181)	0.156 (0.136, 0.178)
Low	0.154 (0.134, 0.171)	0.151 (0.132, 0.172)	0.149 (0.129, 0.176)
Last	0.143 (0.123, 0.167)	0.142 (0.124, 0.163)	0.136 (0.117, 0.158)
b) Squared returns			
	No regressors	An intercept	A linear time trend
Open	0.106 (0.088, 0.126)	0.107 (0.089, 0.127)	0.103 (0.085, 0.124)
High	0.098 (0.078, 0.121)	0.099 (0.080, 0.122)	0.094 (0.074, 0.118)
Low	0.098 (0.079, 0.120)	0.099 (0.080, 0.121)	0.096 (0.077, 0.118)
Last	0.106 (0.088, 0.128)	0.109 (0.090, 0.130)	0.102 (0.082, 0.124)

The values in parentheses give the 95% confidence band for the non-rejection values of d.

Table 3: Estimates of the fractional differencing parameter based on white noise u_t

	No regressors	An intercept	A linear time trend
a) 2 minutes			
Open	0.994 (0.956, 1.038)	0.980 (0.939, 1.028)	0.980 (0.940, 1.028)
High	0.994 (0.956, 1.038)	1.034 (0.989, 1.087)	1.034 (0.989, 1.087)
Low	0.995 (0.957, 1.039)	1.062 (1.017, 1.116)	1.062 (1.017, 1.115)
Last	0.994 (0.957, 1.039)	0.989 (0.948, 1.035)	0.989 (0.949, 1.035)
b) 3 minutes			
	No regressors	An intercept	A linear time trend
Open	0.992 (0.946, 1.047)	0.962 (0.912, 1.019)	0.963 (0.914, 1.019)
High	0.992 (0.946, 1.047)	1.003 (0.950, 1.066)	1.003 (0.951, 1.065)
Low	0.993 (0.947, 1.048)	1.041 (0.984, 1.108)	1.041 (0.985, 1.107)
Last	0.992 (0.946, 1.048)	0.958 (0.907, 1.016)	0.958 0.910, 1.016)
c) 5 minutes			
	No regressors	An intercept	A linear time trend
Open	0.990 (0.930, 1.064)	0.941 (0.872, 1.024)	0.942 (0.877, 1.024)
High	0.990 (0.931, 1.064)	0.948 (0.880, 1.030)	0.949 (0.885, 1.030)
Low	0.990 (0.931, 1.064)	0.981 (0.910, 1.069)	0.982 (0.913, 1.068)
Last	0.989 (0.930, 1.063)	0.942 (0.874, 1.024)	0.944 (0.879, 1.023)
d) 10 minutes			
	No regressors	An intercept	A linear time trend
Open	0.977 (0.895, 1.088)	0.831 (0.719, 0.957)	0.848 (0.761, 0.961)
High	0.978 (0.895, 1.089)	0.869 (0.766, 0.990)	0.881 (0.794, 0.991)
Low	0.977 (0.895, 1.088)	0.860 (0.750, 0.987)	0.873 (0.784, 0.988)
Last	0.978 (0.895, 1.089)	0.861 (0.755, 0.983)	0.872 (0.785, 0.985)

The values in parentheses give the 95% confidence band for the non-rejection values of d.

Table 4: Estimates of d for the absolute returns

a) 2 minutes			
	No regressors	An intercept	A linear time trend
Open	0.188 (0.157, 0.225)	0.182 (0.153, 0.217)	0.179 (0.149, 0.215)
High	0.181 (0.152, 0.216)	0.179 (0.151, 0.211)	0.174 (0.146, 0.208)
Low	0.176 (0.148, 0.210)	0.171 (0.144, 0.202)	0.168 (0.141, 0.200)
Last	0.143 (0.116, 0.173)	0.140 (0.116, 0.169)	0.136 (0.111, 0.166)
b) 3 minutes			
	No regressors	An intercept	A linear time trend
Open	0.159 (0.124, 0.202)	0.157 (0.124, 0.197)	0.151 (0.116, 0.192)
High	0.178 (0.143, 0.221)	0.176 (0.143, 0.216)	0.171 (0.136, 0.212)
Low	0.165 (0.127, 0.212)	0.159 (0.124, 0.202)	0.156 (0.120, 0.200)
Last	0.167 (0.131, 0.211)	0.168 (0.135, 0.210)	0.160 (0.124, 0.204)
c) 5 minutes			
	No regressors	An intercept	A linear time trend
Open	0.189 (0.140, 0.247)	0.176 (0.133, 0.231)	0.175 (0.131, 0.230)
High	0.194 (0.144, 0.259)	0.190 (0.144, 0.249)	0.186 (0.138, 0.247)
Low	0.216 (0.165, 0.281)	0.203 (0.157, 0.262)	0.202 (0.155, 0.261)
Last	0.149 (0.106, 0.204)	0.150 (0.109, 0.202)	0.144 (0.102, 0.198)

Figure 7: Estimates of d for the OPEN series for different bandwidth parameters

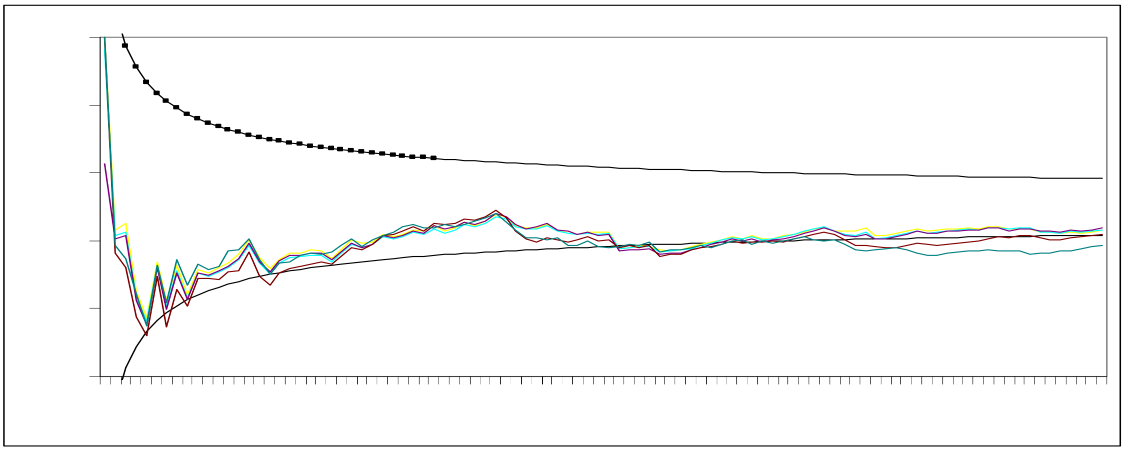
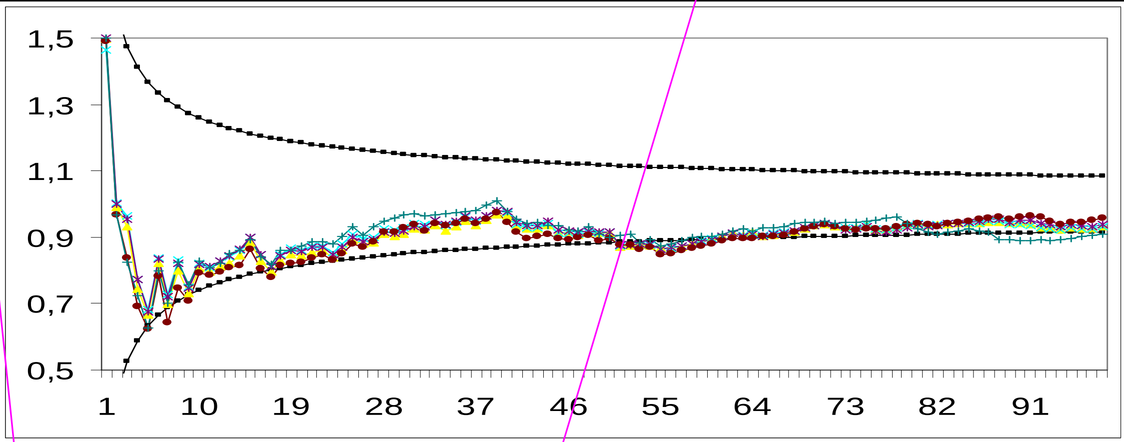


Figure 8: Estimates of d for the HIGH series for different bandwidth parameters



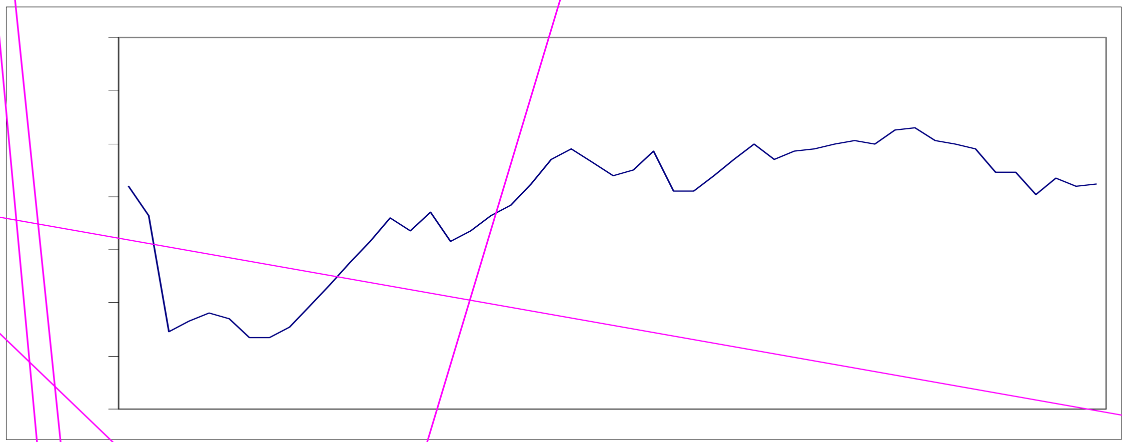


Figure 9: Estimates of d for the LOW series for different bandwidth parameters

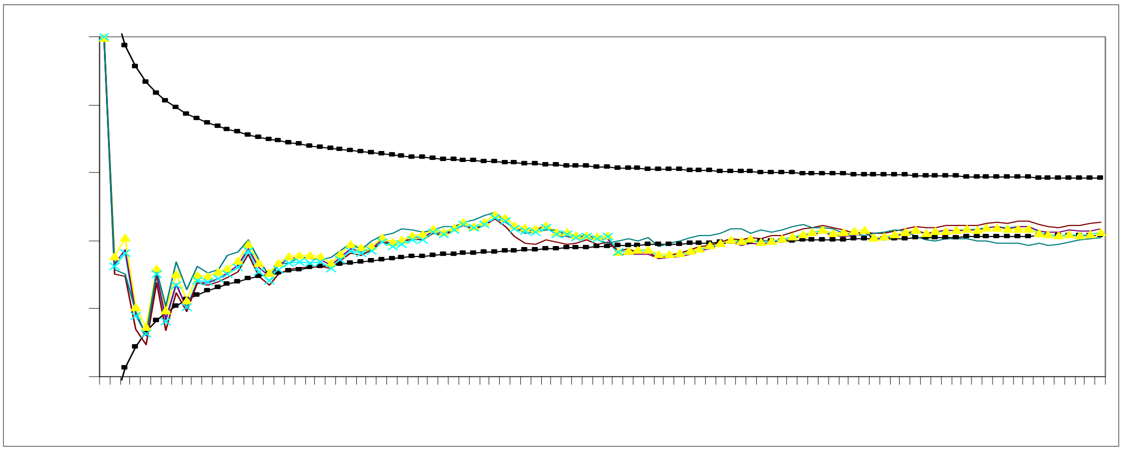


Figure 10: Estimates of d for the LAST series for different bandwidth parameters