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**THE PPP HYPOTHESIS REVISITED:
EVIDENCE USING A MULTIVARIATE LONG-MEMORY MODEL**

**Guglielmo Maria Caporale
Brunel University, London, United Kingdom**

**Luis A. Gil-Alana
University of Navarra
ICS, The Navarra Center for International Development, Pamplona, Spain**

and

**Yuliya Lovcha
University of Navarra, Pamplona, Spain**

January 2013

Abstract

This paper examines the PPP hypothesis analysing the behaviour of the real exchange rates vis-à-vis the US dollar for four major currencies (namely, the Canadian dollar, the euro, the Japanese yen and the British pound). An innovative approach based on fractional integration in a multivariate context is applied to annual data from 1970 to 2011. Long memory is found to characterise the Canadian dollar, the British pound and the euro, but in all four cases the results are consistent with the relative version of PPP.

Keywords: PPP, long memory, multivariate fractional integration

JEL Classification: C22, F31

Corresponding author: Professor Guglielmo Maria Caporale, Centre for Empirical Finance, Brunel University, West London, UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. *Email:* Guglielmo-Maria.Caporale@brunel.ac.uk

1. Introduction

2. Methodology

A covariance stationary process $\{x_t\}$ whose spectral density function is positive and bounded at all frequencies. This includes the class of stationary and invertible ARMA processes, which are characterised by an impulse response function decaying exponentially to zero. On the other hand, the unit root or I(1) class of models require first differences to render the series I(0) stationary and in this case shocks have permanent effects. In between, the I(d, $0 < d < 1$) class of models are mean-reverting but display long-memory behaviour. This implies that the impulse responses decay hyperbolically to zero.

A process $\{x_t\}$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots \quad (1)$$

where x_t is a scalar and u_t is a white noise process. Note that the polynomial $(1 - L)^d$ in (1) can be expressed in terms of its Binomial expansion, such that, for all real d:

time series the spectral density increases dramatically as the frequency approaches zero. However, differencing the data frequently leads to over-differencing at the zero frequency. Robinson (1978) and Granger (1980) then showed that the aggregation of heterogeneous processes could be a source of fractional integration. Since then, fractional processes have been widely employed to describe the dynamics of many economic and financial time series (see, e.g. Diebold and Rudebusch, 1989; Sowell, 1992a; Baillie, 1996; Gil-Alana and Robinson, 1997; etc.) using different univariate procedures. However, univariate methods do not take into account the potential cross-dependence of the series.

The multivariate methodology employed in this paper addresses this issue. The fractionally integrated vector autoregressive model (FIVAR or VARFIMA) can be written as:

$$D L X_t = v_t \quad (3)$$

$$I - F_p L v_t = w_t \quad (4)$$

where X_t is a $N \times 1$ vector of variables for $t = 1, \dots, T$, L is the lag operator, I is an $N \times N$ identity matrix and w_t is an $N \times 1$ vector of i.i.d errors with 0 mean and $N \times N$ variance-covariance matrix Ω . The VAR(p) process in (4) is assumed to be stationary. $D L$ is a diagonal $N \times N$ matrix with fractional integration polynomials on the main diagonal given by (2).

To estimate the process given by (3) and (4) we use the approximate frequency domain maximum likelihood (Whittle) estimator proposed by Boes et al. (1989). A discussion of the multivariate version of this procedure can be found in Hosoya (1996).

pound). The method used is more flexible than standard tests and takes into account possible cross-dependence.

